Lesson 2

Time Value of Money

Objectives of the lesson

After studying this lesson, students will be able to:

- Explain the concept and rationale of time value of money,
- Understand the components of time value of money,
- Calculate future value of single cash flow and annuity,
- Explain the concept of doubling period, and effective and nominal rate of interest, and
- Calculate present value of single cash flow and annuity.

1.0 Introduction

A finance manager is required to make decisions on investment, financing and dividend keeping in view the objectives of the company. The investment and financing decisions such as purchase of assets or procurement of funds affect the cash flows in different time period. For example, if a fixed asset is purchased it will require cash outflow immediately and cash inflows will generate over a period of time. Similarly, in the case of borrowing from bank cash is received immediately and it is required to be repaid over a period of time. These cash inflows and outflow at different point of time are not comparable because a rupee received now is not comparable with a rupee to be received in future. However, they can be made comparable by introducing the interest factor, a crucial and exclusive concept, known as time value of money.

2.0 Time Value of Money: Meaning and Rationale

The time value of money is a basic financial concept that holds that money in the present is worth more than the same sum of money to be received in the future. It has gained importance in studying the viability of the project by comparing the initial investment with the anticipated future benefits. If the anticipated future benefits are more than the initial investment, then the investment is found to be viable in generating the economic benefits. The main reasons of time preference of money are:

- There is uncertainty about the receipt of money in future.
- Most of the persons and the companies have preference for present consumption then future consumption either because of urgency of need e.g. consumer durable or otherwise.
- Most of the persons and the companies have preference for present money because of availabilities of opportunities of investment for earning additional cash flows.

- In an inflationary period, a rupee today has greater purchasing power than rupee in the future. So, they want to find the real rate of return with reference to money employment in productive assets.

3.0 Components of Time Value of Money

Time value of money normally consists three components, viz., (i) real rate of return, (ii) expected or anticipated rate of return, and (iii) risk premium. Real rate of return is the return which considers original return on investment, but it never considers the inflation rate. Expected rate of return is the positive rate of return normally expected by everyone on the amount of investment from the future. And risk premium is an allowance, normally given to investors to compensate the uncertainty.

The concept of time value of money can be classified into two major categories, (i) Future value of money, i.e., compounding, and (ii) Present value of money, called as discounting.

4.0 Future Value of Money (Process of Compounding)

Under the method of compounding, we find the future values (FV) of all the cash flows at the end of the time horizon at a particular rate of interest. Various intricacies of future value of money are discussed below.

4.1 Future Value of Single Cash Flow: A generalized procedure (formula) for calculating the future value of a single cash flow compounded annually is:

$$FV_n = PV\,(I + k)^n$$

Here: FV_n = Future value of the initial flow n years hence, PV = Initial cash flow, k = Annual rate of interest, n = Life of investment.

In the above formula, the expression $(I + k)^n$ represents the future value of an initial investment of Re.1 (one rupee invested today) at the end of n years at a rate of interest k referred to as Future Value Interest Factor (FVIF, hereafter). To simplify calculations, this expression has been evaluated for various combinations of k and n and these values are presented in Table at the end of this book. To calculate the future value of any investment for a given value of 'k' and 'n', the corresponding value of $(I + k)^n$ from the table has to be multiplied with the initial investment.

4.2 Future Value of Single Cash Flow (Case of Increased Frequency of Compounding): Increased frequency of compounding can be understood with the help of an example.

For example, a bank offers 10% interest per annum compounded semi-annually then it means interest is paid every six months. In this case FVncan be calculated by the formula:

$$FVn = PV\left(1 + \frac{k}{m}\right)^{m \times n}$$

Here: FVn = Future value after n years, PV = Cash flow today, k = Nominal interest rate per annum, m = Number of times compounding is done during a year, n = Number of years for which compounding is done.

4.3 Future Value of Annuity: Annuity is the term used to describe a series of periodic flows of equal amounts. These flows can be either receipts or payments. For example, if an investor is required to pay Rs. 2,000 per annum as life insurance premium for the next 20 years, he can classify this stream of payments as an annuity.

When cash flows occur at the beginning of each period the annuity is known as an annuity due. The future value of a annuity due for a period of n years at given rate of interest 'k' can be calculated by the following formula.

$$\text{FVA n} = A\left(\frac{(1+k)^n - 1}{k}\right)(1+k)$$

Here: FVA_n = Accumulation at the end of n years, A = Amount deposited in the beginning of every year for n years, k = Rate of interest and n = Time period

If the equal amounts of cash flow occur at the at the end of each period over the specified time, this stream of cash flows is defined as a regular annuity or ordinary annuity or deferred annuity. It can be calculated as:

$$FVA_n = A\,(I + k)^{n-1} + A\,(I + k)^{n-2} + A\,(I + k)^{n-3} + \ldots\ldots\ldots$$

This can also be written as:

$$\text{FVA n} = A\left(\frac{(1+k)^n - 1}{k}\right)$$

Here: FVA_n = Accumulation at the end of n years, A = Amount deposited at the end of every year for n years, k = Rate of interest and n = Time period

5.0 Doubling Period

Investors frequently ask a very common question, "How long will it take for the amount invested to be doubled at a given rate of interest". This question can be answered by 'Rule of 72' and 'Rule of 69.

According to Rule of 72:

$$\text{Doubling Period} = \frac{72}{k}$$

According to Rule 69:

$$\text{Doubling Period} = 0.35 + \frac{69}{k}$$

An accurate way of calculating doubling period is the 'Rule of 69'.

6.0 Effective and Nominal Rate of Interest

The future value in schemes in which compounding is done more than once in a year, exceeds the accumulation under the annual compounding schemes. It means if explicit rate of interest (called as Nominal Rate of Interest) is 10%, the amount in annual compounding will grow at 10% per annum, while under the scheme where compounding is done half yearly or quarterly basis, the principal amount will grow at the rate more than 10% per annum. This rate is called as Effective Rate of Interest. The effective rates of interest can be calculated as:

$$r = \left(1 + \frac{k}{m}\right)^{m} - 1$$

Here: r = Effective rate of interest, k = Nominal rate of interest, m = Frequency of compounding per year

7.0 Present Value of Money (Process of Discounting)

Under the method of discounting, we reckon the time value of money now i.e. at time 0 on the timeline. Here, we compare initial outflow with the sum of the present values (PV) of future cash inflows at a given rate of interest. The calculation of present value of money has two cases, (i) Present Value of a Single amount, and (ii) Present Value of an Annuity.

7.1 Present Value of Single Amount: The present value of future cash flows (single amount) can be calculated by the formula:

$$\mathrm{PV} = \left(\frac{\mathrm{FVn}}{(1+\mathrm{k})^{\mathrm{n}}}\right) or\ \mathrm{PV} = \mathrm{FVn}\left(\frac{1}{(1+\mathrm{k})^{\mathrm{n}}}\right)$$

7.2 Present Value of an Annuity: The present value of an annuity receivable at the end of every year for a period of n years at a given rate of interest is equal to:

$$\mathrm{PVA\ n} = \left(\frac{\mathrm{A}}{(1+\mathrm{k})^{1}}\right) + \left(\frac{\mathrm{A}}{(1+\mathrm{k})^{2}}\right) \ldots + \left(\frac{\mathrm{A}}{(1+\mathrm{k})^{\mathrm{n}}}\right)$$

This equation in reduced form can be written as:

$$\mathrm{PVA\ n} = \mathrm{A}\left(\frac{(1+\mathrm{k})^{\mathrm{n}} - 1}{\mathrm{k}(1+\mathrm{k})^{\mathrm{n}}}\right)$$

It must be noted that these values can be used in any present value problem only if the following conditions are satisfied: (a) the cash flows are equal; and (b) the cash flows occur at the end of every year.

Summary

Inflation, uncertainty and opportunity cost, whatever may be the reason, money has time value. A rupee today is certainly more valuable than a rupee a year hence, the difference usually represented by interest. Therefore, two cash flows occurring at different points of time are not comparable. Compounding and discounting are two methods used to take care of time value of money. Discounting involves determining the present values of all the future cash flows so that they are comparable to the initial outflow. The rate of interest usually employed is the cost of capital of the firm.

Self Check Questions

1. What is the relevance of time value of money in financial decision making? Explain.
2. Explain the discounting and compounding techniques of time value of money.
3. Write detailed note on: (a) Doubling period, (b) Nominal and Effective Rate of Interest, (c) Present Value of Annuity, and (d) Future Value of Annuity
4. Attempt related practical problems by referring suggested readings.

Practical Problems on Time Value of Money

Illustration 1: The fixed deposit scheme of a bank offers 11% interest rate for 3 years. Find the maturity value of FD after 3 years.

Solution: $FV_n = PV\,(I + k)^n$

$FV_n = 10{,}000\,(I + 11)^3$

$FV_n = 10{,}000\,(1.368) =$ Rs.13, 680

Illustration 2: An investor deposited Rs. 1,000 in a scheme for 2 years. The scheme offers 10% interest with quarterly compounding. Find the maturity value of the scheme.

Solution: By formula:

$$FVn = 1{,}000\left(1 + \frac{0.10}{4}\right)^{4\times 2}$$

$FVn = 1{,}000\,(1.025)^8 =$ Rs. 1,218

Illustration 3: A person is required to pay 4 equal annual payments of Rs. 5,000 each in his deposit account that pays 8% interest per year. Find out the future value of annuity at the end of 4 years.

Solution:

$$FVAn = 5{,}000\left(\frac{(1 + 0.08)^4 - 1}{0.08}\right)$$

$FVAn =$ Rs. $5{,}000 \times 4.507 =$ Rs. 22, 535

Illustration 4: If the rate of interest is 8 percent, find the doubling period.

Solution:

$$\text{By Rule of 72: Doubling Period} = \frac{72}{8} = 9 \text{ Years}$$

$$\text{By Rule 69: Doubling Period} = 0.35 + \frac{69}{8} = 8.66 \text{ Years}$$

Illustration 5: Find out the effective rate of interest, if the nominal rate of interest is 12% and interest is quarterly compounded.

Solution:

$$r = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$r = (1 + 0.03)^4 - 1$

$r = 1.126 - 1 = 0.126 = 12.6\%$ p.a.

Illustration 6: A company offers a bond for a period of 5 years having redemption value of Rs. 1,611. Prevailing rate of interest is 10%. Find the present value the bond.

Solution:

$$\mathrm{PV} = \left(\frac{1{,}611}{(1+0.10)^5}\right)$$

$$\mathrm{PV} = \mathrm{Rs.}\ 1,\ 000$$

Illustration 7: A bank is planning to offer deposit certificates under reinvestment plan having redemption value of Rs. 100 and interest @12%. Interest on deposit earns, is as it is reinvested at quarterly rests. What should be the issue price of these certificates?

Solution: Since interest is reinvested at quarterly rests, we will have to calculate effective rate of interest.

$$\mathrm{r} = \left(1+\frac{0.12}{4}\right)^4 - 1 = 0.1255 \text{ i.e.}, 12.55\%$$

The issue price of the cash certificate should be:

$$\mathrm{PV} = \left(\frac{100}{(1+0.12)^1}\right) = \mathrm{Rs.}\, 88.55$$

Illustration 8: Bank of Baroda is offering a deposit scheme in which a lump sum deposit of certain sum of amount will give a monthly return of Rs. 100 (principle and interest). The interest is compounded at quarterly intervals. Calculate the offer price (present value) of the deposit scheme.

Solution: The amount of initial deposit to receive a monthly installment of Rs.100 for 12 months can be calculated as below:

Firstly, we will have to calculate annual effective rate of interest.

$$\mathrm{r} = \left(1+\frac{0.12}{4}\right)^4 - 1 = 0.1255 \text{ or } 12.55\%$$

After calculating the effective rate of interest per annum, the effective rate of interest per month has to be calculated. This is:

$$(1.1255)^{\frac{1}{12}} - 1 = 0.0099$$

The initial deposit can now be calculated as:

$$\mathrm{PVAn} = 100\left(\frac{(1+0.0099)^{12} - 1}{0.0099(1+0.0099)^{12}}\right)$$

$$\mathrm{PVAn} = 100 \times \frac{0.1255}{0.01114} = 100 \times 11.26 = \mathrm{Rs.}\, 1{,}126$$