Lesson 9

Techniques of Capital Investment Decisions under Uncertainty

Objectives of the lesson

After studying this lesson, students will be able to:

- Understand techniques of capital budgeting under uncertainty, and
- Learn applications of these techniques in investment decisions.

1.0 Introduction

Since future estimates are exposed to different kinds of risk, forecasts cannot be made with certainty. Risk arises in investment evaluation because it is difficult to anticipate the future events with certainty. Cash flows depend on future events and these are likely to change. A project may be evaluated because it has a very high demand in the present. But this would be different if there are shifts in the future demand of a product due to economic and political changes or customers preferences. Risk and profitability are closely associated. A profitable project may also be very risky. A financial manager has to manage a trade-off between risk and return. The risks associated with investment decisions may include economy risk, market risk, industry specific risk, unique risks concerned with a particular company, and risks associated with international political and economic environment.

2.0 Techniques of Capital Budgeting under Uncertainty

A number of techniques have been developed to handle risk in capital budgeting. These techniques consist of the conventional techniques such as (i) Payback technique, (ii) Risk adjusted discount rate, (iii) Certainty equivalent approach, (iv) Sensitivity analysis; and statistical techniques such as (i0 Probability distribution approach and (ii) Decision tree approach.

2.1 Pay-back Period

The payback period is used mainly for finding out the time period when the investment is recovered by the firm. If the payback period is less than the standard expected payback period, the proposal is selected. It is a very simple method and is quite popular with business firms because they prefer short recovery periods of their investments. The payback period method provides an indication whether the projects have a short or long recovery period.

The payback period focuses on recovery of the investment. As a technique of risk analysis, it is useful in assessing the risks of time nature, and it ignores the time value of money. For example, it if there are two prefects with a 5-year payback and project A recovers its capital with even cash flows every year, it will be preferred to project B which recovers its capital in the last year. The project B will be considered more risky. The payback period method fails to measure the risk, which differs in different kind of projects. It also does not consider the cash flows after the payback period. The payback period method can be used after the risk has been identified by: (i) adjusting the discount rate, and (ii) using a risk less rate of discount.

2.1 Risk Adjusted Discount Rate (RADR)

The amount of risk in the project is inbuilt in the discount rate to make the present value calculations. The discount rate would be high when the risk is high. For example, a discount rate of 10% may be considered useful to calculate the cash inflows of a project. In order to incorporate risk, the discount rate can be increased. If it is low risk, it can be increased to 11% and if it is high risk the discount rate can be increased to a higher rate. In this case as the risk goes on increasing the discounting rate can be 12% or 13% or even 15% if the risk is very high. The difference between a proposal, which is at a risk-less rate, and a risky one is called the risk premium, or the risk adjusted discount rate. The risk adjusted discount rate considers both time and risk factors. This risk can be measured by using statistical measures like standard deviation and coefficient of variation. It can also be calculated by using through the net present value method. The NPV formula is given as.

$$\mathrm{NPV} = \sum \frac{\mathrm{CF_t}}{(1+\mathrm{K})^{\mathrm{t}}} - \mathrm{C_o}$$

Here, CF = Expected cash inflows, k = Adjusted discount rate, and C_0 = Initial cash outflow

The accept/reject decision through the RADR technique can be used both in NPV and IRR calculations. If NPV is used, the cash inflows should be higher than the cash outflows, at the adjusted discount rate, to be accepted as a good capital budget proposal. In the case of IRR, the internal rate of return should exceed the risk adjusted rate of return.

2.3 Certainty Equivalent Approach (CE)

This approach is considered to be an alternate method to the RADR approach. It considers the adjustments of cash inflows and not the discount rate. This is based on the fact that the future returns are uncertain and there is a risk in receiving future cash inflows. The method of applying this technique is to identify the cash inflows and then to incorporate risk into it.

There are certain statistical methods also which enable the firms to take decisions under risky conditions. Let us now give an introduction to statistical techniques, which are used to incorporate risk in capital budgeting decisions.

2.4 Sensitivity Analysis

This approach helps to take decisions of accepting or rejecting a project through the estimation of Net Present Value. If three situations are given when cash flow estimates are provided, the Net Present Value (NPV) will decide which of the project should be selected. Sensitivity analysis are also evaluated in conditions of finding out about a project when there is a change in one of the variables by analyzing after tax cash flows of combined forecast of the variables. The sensitivity analysis can be used through NPV or IRR. The steps for analyzing sensitivity analysis are: Identify the variables, which have an influence on the NPV or IRR of a proposal; the mathematical relationship of the variable should be defined; an analysis of changes in the variable is made through the NPV; and on the basis of the results the proposal may be accepted or rejected.

2.5 Probability Distribution Approach

Capital budgeting decisions are made by forecasting future cash flows. There are uncertainties in assigning these cash flows. Where cash flows are independent in nature and the cash flows receipt in the current year do not have any relationship with the next year, probabilities can be assigned to the cash flows. Probability is a measure, which is related to the likelihood of the occurrence of an event. If a probability certainly occurs it is said that there is the probability of one, but when it is not certain the probability of occurring of the event is zero. Thus probabilities lie between 0 and 1. A probability distribution consists of many estimates but usually firms/companies have three or four estimates. For examples, a probability distribution can be classified as- optimistic, likely and pessimistic. Similarly it can be classified into high risk, medium risk and low risk. The annual cash flow from project can be classified into Rs. 3,50,000 as optimistic; Rs. 2,00,000 as likely and Rs. 1,00,000 as pessimistic estimates of the future cash flow. To these estimates probability of the occurrences will be assigned to make it more accurate.

Table Depicting Situation Cash blows and Probabilities

Situation	Cash flows (Rs.)	Probability
Optimistic	3.50,000	0.20
Likely	2.00.000	0.60
Pessimistic	1.00.000	0.20

After assigning probabilities the expected net present value will be calculated. The net present value is found out by multiplying the monetary values of the cash flows with their probability. The total of the net present value is calculated with the following formula:

$$\mathrm{NPV} = \sum \frac{\mathrm{CF_t}}{(1+\mathrm{K})^{\mathrm{t}}} - \mathrm{C_o}$$

Risk is incorporated in capital budgeting decisions through the calculation of the sum of the net present values. With the net present value technique, the standard deviation is combined to measure the dispersion or difference in the possible cash flows, which are estimated. When two proposals are compared, standard deviation will indicate which one of the projects is more risky in absolute terms. This sometimes creates a problem in choice of a project because a project may be profitable, but it may also be more risky. In such a case, coefficient of variation is used as it is a relative measure of risk.

$$\sigma(\mathrm{NPV}) = \sqrt{\frac{{\sigma_1}^2}{(1+\mathrm{i})^2} + \frac{{\sigma_2}^2}{(1+\mathrm{i})^4} + \frac{{\sigma_3}^2}{(1+\mathrm{i})^6}} = \sqrt{\sum \frac{{\sigma_t}^2}{(1+\mathrm{i})^{2t}}}$$

For calculating probability of the NPV being zero or less following formula is used.

$$\text{Normal Variety (Z)} = \frac{\mathrm{x} - \bar{\mathrm{x}}}{\sigma}$$

2.6 Decision Tree Approach

In the previous technique, we have discussed the probability distribution approach. In that technique, evaluation was useful, when cash flows from one year to the next were independent of each other. The decision tree approach is useful when the cash flows of the project are dependent in nature. In this method cash flows are assigned probabilities. Every future possibility in terms of cash flow is evaluated after giving them weights in probability terms. It takes the impact of all potential cash flows. Decision tree approach evaluates risky investments. It can also evaluate a sequential part to decisions. It is a representation of cash flows like a picture. It is called a decision tree because it has a structure, which looks like a tree with branches.

Summary

There are different methods for evaluating risk and capital decisions. These are discounted payback method, RADR method, CE method, sensitivity analysis, probability distribution and decision tree. Risk is the variability of return of a project. If it is not evaluated or estimated, the estimation of future cash flows will not give the soundness of the project. In the RADR approach adjusting the discount rate incorporates risk. The risky projects have a higher discount rate than the projects with lower risk. The CE approach shows the relationship between risky and risk-less cash flows. The discounted cash flow method provides discount rate to the payback period. The probability distribution approach is able to analyze risk through probabilities. The decision tree approach estimates potential outcomes. It also takes into account the impact of probable estimates.

Review Questions

1. Discuss the advantages of risk adjusted discount rate. What is the problem in using this approach?
2. 'Certainty equivalent approach is theoretically superior to the risk adjusted discount rate'. Do you agree? Give reasons.
3. What are the limitations of the payback method to handle risk?
4. Write notes on: (i) Certainty Equivalent method (ii) Probability Distribution method (iii) Sensitivity Analysis (iv) Decision Tree.
5. What is Decision Tree? How is it different from the Probability Distribution Method?
6. How is risk handled by using the Probability Distribution Method? Explain clearly.

Practical Problems on Capital Investment Decisions under Uncertainty

Illustration # 1: The proposal has a cash outlay of Rs. 5, 00,000. It is expected to generate cash flows of Rs. 2, 50,000, Rs. 2, 00,000, Rs. 1, 00,000 and Rs. 1, 00,000 respectively from year l to year 4. It had a risk free discount rate of 8% and a lifetime of 4 years. However, due to increase in risk, adjusted discounted rate of 10% is applied. Find out if the proposal should be accepted.

Solution:

Year	Cash Inflows	PVF@10%	PVCF
1	2,50.000	0.909	2,27.250
2	2,00.000	0.826	1,65,200
3	1,00.000	0.751	75,100
4	1,00,000	0.683	68,300
	Sum of PV of Cash Inflows		5,35,850
	Cash Outflows		5,00,000
	NPV		35,850

The proposal at the higher risk rate of 10% will be acceptable

Illustration # 2: X Ltd. is considering a new project for which it has gathered following data:

NPV	80.000	1,10,000	1,42,500
Probability	0.3	0.3	0.2

Compute the risk associated with the project i.e. standard deviation.

Solution:

NPV	Probability	Expected NPV
80,000	0.3	24,000
1,10,000	0.3	33,000
1,42,500	0.2	28,500
Average NPV		85,500

Calculation of standard deviation of Project:

NPV	D	D^2	P	PD^2
80,000	- 5,500	3,02,50,000	0.3	90,75,000
1,10,000	24,500	60,02,50,000	0.3	18,00,75,000
1,42,500	57,000	3,24,90,00,000	0.2	64,98,00,000
σ^2 =				83,89,50,000
σ =				28,965
Coefficient of variation = σ/ NPV =				28,965 85,500 = 0.34

Illustration # 3: A company is considering Projects X and Y with following information:

Project	Expected NPV (Rs.)	Standard deviation
X	18,000	6,500
Y	22,000	7,200

Which project will you recommend?

Solution: On the basis of information about standard deviation of Project X & Y, the Project X is better as it has lower standard deviation (i.e. risk). However, the coefficient of variation for these projects may be found as follows.
Coefficient of variation = σ/ NPV
Project X = 6,500/ 18,000 = 0.361
Project Y = 7,200/ 22,000 = 0.327
Project Y is better as its CV is lesser than Project X.

Illustration # 4: A firm estimates an inflow of cash of Rs. 15, 00,000 per annum, but this has a risk. The expectation is that the cash inflows of Rs. 10, 00,000 will be certain. Find out the certainty equivalent (CE) coefficient.

Solution:

The CE coefficient would be the relationship between risk-less and risky cash flows or risk-less cash flows/ risky cash flows or Rs. 10,00,000/15,00,000 = 0.6 x (i.e., Risky cash inflow) x Rs. 15,00,000 = Rs. 9,00,000 (Certainty cash flow). The coefficient can assume a value between zero and one and can be used by both NPV and IRR techniques.

Illustration # 5: There are two proposals. The following details pertain to them. Which one should be selected?

Proposal	Cash Flows (Rs)		
	Pessimistic	Likely	Optimistic
1	8,00,000	10,00,000	16,00,000
2	2,00,000	10,00,000	20,00,000

The cost of capital is 14% and cash flow annuity estimate are of 10 years. The cost of the proposals is 50,00,000 lakh. Calculate NPV.

Solution

Proposal 1

Situation	CFAT	Annuity @14%	PV	Cost	NPV
Pessimistic	8,00,000	5.216	41,72,800	50,00,000	-8,27,200
Likely	10,00,000	5.216	52,16,000	50,00,000	2,16,000
Optimistic	16,00,000	5.216	83,45,600	50,00,000	33,45,600

Proposal 2

Situation	CFAT	Annuity @14%	PV	Cost	NPV
Pessimistic	2,00,000	5.216	10,42,200	50,00,000	-39,57,200
Likely	10,00,000	5.216	52,16,000	50,00,000	2,16,000
Optimistic	20,00,000	5.216	1,04,32,000	50,00,000	54,32,000

Proposal 2 is riskier than proposal 1 because it has higher negative value in pessimistic situation. Choose proposal 1as per sensitive analysis.

Illustration # 6: A project has a cash outlay of Rs. 6, 00,000. It is expected to generate the following cash flows with associated probabilities in next 3 years.

Period 1		Period 2		Period 3	
CFAT	Probability	CFAT	Probability	CFAT	Probability
3.50.000	0.1	3,00,000	0.3	3,00,000	0.2
2.00,000	0.2	2,75,000	0.2	2,00,000	0.2
4.00.000	0.3	3,50,000	0.2	3,00,000	0.3
3.00.000	0.4	3,00,000	0.3	2,00,000	0.3

Firm's cost of capital is 10%. Risk free rate is 5%. Determine the expected value of NPV and standard deviation about the expected value.

Solution

Step 1: Determination of Expected NPV

Period 1			Period 2			Period 3		
CFAT	P		CFAT	P		CFAT	P	
3.50.000	0.1	35.000	3,00,000	0.3	90.000	3,00,000	0.2	60.000
2.00,000	0.2	40,000	2,75,000	0.2	55,000	2,00,000	0.2	40,000
4.00.000	0.3	1.20.000	3,50,000	0.2	70.000	3,00,000	0.3	90.000
3.00.000	0.4	1.20.000	3,00,000	0.3	90.000	2,00,000	0.3	60.000
Av. CF		3,15,000	Av. CF		3,05,000	Av. CF		2,50,000

Step 2: Determination of expected NPV of period 1, 2, 3

Average CF	PVF@5%	PV of Average CF
3,15,000	0.952	2,99,880
3,05,000	0.907	2,76,635
2.50,000	0.864	2,16,000
	Sum of PV	7,92,515
	Cash Outflows	6,00,000
	Expected NPV	1,92,515

Step 3: Determination of Standard Deviation

CF – Av. CF	$(CF - Av. CF)^2$	Prob.	$(CF - Av. CF)^2$ x P
Period 1: Average CF = 3,15,000			
CF – Av. CF	$(CF - Av. CF)^2$	Prob.	$(CF - Av. CF)^2$ x P
35,000	1,22,50,00,000	0.1	12,25,00,000
-1,15,000	13,22,50,00,000	0.2	2,64,50,00,000
85,000	7,22,50,00,000	0.3	2,16,75,00,000
-15,000	22,50,00,000	0.4	9,00,00,000
$\sum(CF - Av. CF)^2$ x P			5,02,50,00,000
$\sigma = \sqrt{5,02,50,00,000}$			70,887.2
Period 2: Average CF = 3,05,000			
CF – Av. CF	$(CF - Av. CF)^2$	Prob.	$(CF - Av. CF)^2$ x P
5,000	2,50,00,000	0.3	75,00,000
30,000	90,00,00,000	0.2	18,00,00,000
45,000	2,02,50,00,000	0.2	40,50,00,000
5,000	2,50,00,000	0.3	75,00,000
$\sum(CF - Av. CF)^2$ x P			60,00,00,000
$\sigma = \sqrt{60,00,00,000}$			24,494.9
Period 3: Average CF = 3,50,000			
CF – Av. CF	$(CF - Av. CF)^2$	Prob.	$(CF - Av. CF)^2$ x P
50,000	2,50,00,00,000	0.2	50,00,00,000
-50,000	2,50,00,00,000	0.2	50,00,00,000
50,000	2,50,00,00,000	0.3	50,00,00,000
-50,000	2,50,00,00,000	0.3	50,00,00,000
$\sum(CF - Av. CF)^2$ x P			2,50,00,00,000
$\sigma = \sqrt{2,50,00,00,000}$			50,000.0

Step 4: Determination of Standard Deviation of the probability distribution of NPV

$$\sigma(NPV) = \sqrt{\frac{{\sigma_1}^2}{(1+i)^2} + \frac{{\sigma_2}^2}{(1+i)^4} + \frac{{\sigma_3}^2}{(1+i)^6}}$$

$$\sigma(NPV) = \sqrt{\frac{(70,887.2)^2}{(1+0.1)^2} + \frac{(24,494.9)^2}{(1+0.1)^4} + \frac{(50,000.0)^2}{(1+0.1)^6}}$$

$$\sigma(NPV) = \sqrt{\frac{5,02,50,00,000}{1.210} + \frac{60,00,00,000}{1.464} + \frac{2,50,00,00,000}{1.772}}$$

$$\sigma(NPV) = \sqrt{4,15,28,92,562 + 40,98,36,066 + 1,41,08,35,214}$$

$$\sigma(NPV) = \sqrt{5973563842} = 77,289$$

Step 5: Calculation of probability of NPV being zero or less.

$$Z = \frac{x - \bar{x}}{\sigma}$$

$$Z = \frac{0 - 1{,}92{,}515}{77{,}289} = -2.49$$

If we look at table for Z values, −2.49 is under the required area of 0.5. Therefore, probability of being less than zero is 50%.

Illustration # 7: A company has made following estimates if the CFAT of the proposed project. The company use decision tree analysis to get clear picture of project's cash inflow. The project cost Rs. 80,000 and the expected life of the project is 2 years. The net cash inflows are:

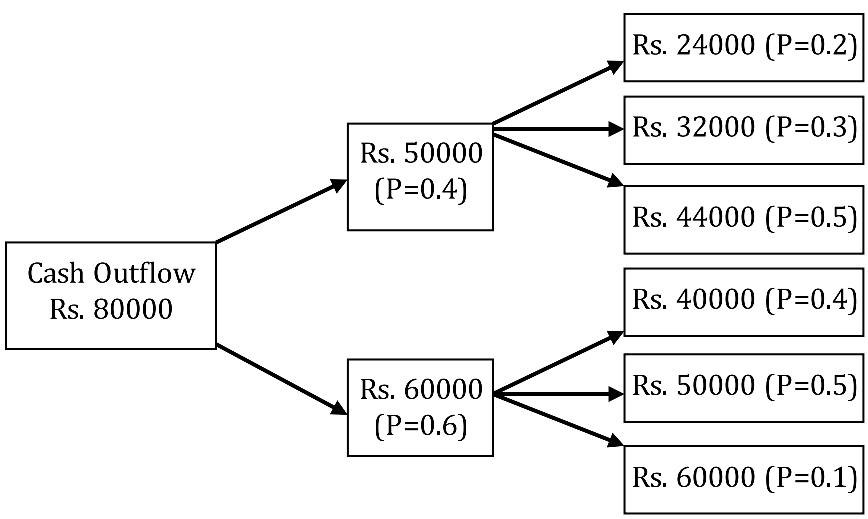
Year 1: There is 0.4 probability that CFAT will be Rs. 50,000 and 0.6 probability that CFAT will be Rs. 60,000.

Year 2: The probabilities assigned to CFAT are as follows.

If CFAT = Rs. 50,000		If CFAT = Rs. 60,000	
Cash Flow	Probability	Cash Flow	Probability
24,000	0.2	40,000	0.4
32,000	0.3	50,000	0.5
44,000	0.5	60,000	0.1

The firm uses 10% discount rate for this type of investments.

Solution: Decision Tree



Net present value of cash flows

Combination	$CFAT_1$	PV Factor	PV_1	$CFAT_2$	PV Factor	PV_2
A	50,000	0.909	45,450	24,000	0.826	19,824
B	50,000	0.909	45,450	32,000	0.826	26,432
C	50,000	0.909	45,450	44,000	0.826	36,344
D	60,000	0.909	54,540	40,000	0.826	33,040
E	60,000	0.909	54,540	50,000	0.826	41,300
F	60,000	0.909	54,540	60,000	0.826	49,560

Table continued

Combination	Total PV ($PV_1 + PV_2$)	Initial Investment	NPV	Joint Probabilities	Expected NPV
A	65,274	80,000	- 14,276	0.08	- 1,178
B	71,882	80,000	- 8,118	0.12	- 974
C	81,794	80,000	1,794	0.20	358
D	87,850	80,000	7,580	0.24	1,819
E	95,840	80,000	15,840	0.30	4,752
F	1,04,100	80,000	24,100	0.06	1,446
				Total	6,223